Commensuration and theories of value in ecological economics

Murray Patterson *

Department of Resource and Environmental Planning, Massey University, Palmerston North, New Zealand

Received 3 February 1997; accepted 30 September 1997

Abstract

Without a theory of value (price) in ecological economics, the valuation of ecosystem and economic resources cannot be rigorously defended. At the crux of the matter is the 'valuation' or 'mixed units' problem of commensurating biophysical inputs/outputs which are different. This paper critically reviews candidate theories of value which have emerged in the ecological economics literature for dealing with this problem. First of all, the Neo-Ricardian approach, using Sraffa-type systems is discussed. In particular, Judson's assertion that the Sraffa system is suitable for ecological economics is questioned, as it is based on 'circular flow', generation of a 'surplus' and 'exchange values'. None of these ideas are consistent with a biophysical characterisation of an economic system, which is paramount in ecological economics. A more fruitful avenue for establishing a biophysical theory of value based on ecological and thermodynamic principles, is to extend Patterson's quality equivalent methodology (QEM) to explicitly include mass flows in addition to energy. At this point a model close to Costanza and Hannon's biosphere model of price determination emerges. The extended QEM model however, has a number of important advantages over the Costanza–Hannon model because it has a more generalised formulation and is not reliant on necessarily using solar energy as the numeraire. The paper concludes with a discussion of some outstanding issues that need to be resolved before a biophysical theory of value can be properly established in ecological economics. © 1998 Elsevier Science B.V. All rights reserved.

Keywords: Biophysical theories of value; Commensuration; Sraffian economics; Mixed units problem

1. Introduction

1.1. Background and purpose of the paper

Theories of value have formed the theoretical core of every major school of economic thought, from the classical era right through to the rise of...
neoclassical economics. Tensions between schools of economic thought still continue to be underpinned by fundamental disagreements over theories of value (Cole et al., 1991). The same can be said about the importance of theories of value with respect to the paradigmatic tensions that exist between ecological economics and neoclassical environmental/resouce economics. The issue of a theory of value (price) has however not received much attention in the ecological economics literature, with the exception of Hannon (1976), Hannon et al. (1986), Costanza and Hannon (1989), Judson (1989) and Amir (1989). It has often been suggested by such authors that an ‘energy theory of value’ is appropriate for ecological economics. This suggestion has given rise to a heated debate with neoclassical economists such as Huettner (1976) and Webb and Ricketts (1980), who assert that energy is only one of the factors of production and that the neoclassical price theory based on a marginal analysis of supply and demand is required.

Theories of value in ecological economics will always set it apart from neoclassical price theory because of the biophysical perspective brought to bear by ecological economists. In this light, Judson (1989) conjectures that the Sraffa (1960) Neo-Ricardian theory of value is more compatible with the emergent energy theories of value in ecological economics. Both the Neo-Ricardian and the energy theory of value are based on physical accounting and on a ‘cost-of-production’ approach, with no recourse to demand and supply schedules specified by consumer preference. The analytical purpose of this paper is to re-examine the Judson (1989) conjecture by critically analysing three candidate theories of value: Sraffa’s (1960) Neo-Ricardian theory of value; Patterson’s (1983, 1993, 1996) embodied energy theory of value and the Costanza and Hannon (1989) biosphere model of price determination. In particular, each candidate theory of value will be scrutinised in terms of a set of biophysical accounting principles which draw upon thermodynamic and ecological theory. The paper concludes with a discussion of the methodological issues that need to be resolved before any ecological theory of value can be established.

1.2. Theories of value in classical and neoclassical economics

It is necessary to provide a brief synopsis of the main theories of value as they have emerged in mainstream economics, as an epistemological backdrop to this paper. In this way, the unique characteristics of the theories of value in ecological economics can be more fully appreciated, as can the commonalities between ecological economics theories of value and other approaches.

The Physiocrats, led by Francois Quesnay (1694–1774), made an early contribution that many contemporary ecological economists draw upon. They based their theorising on a natural (ecological) view of the world, which emphasised natural laws, interdependencies between sectors and circular flow of commodities. ‘Land’ (natural resources) was seen to be the unequivocal source of all value. Within this context, only the agricultural sector was considered to be productive and capable of producing a surplus as it derived all its wealth directly from the land. Manufacturers were seen to be the ‘sterile’ class, as they added no value to the products from the agricultural sector. The Physiocrats used land as the value-numeraire, although they did not construct a rigorous theory of value (price).

Adam Smith (1723–1790), made the most significant early contributions to the theoretical understanding of the concept of value. In essence, Smith presented two theories of value which he had difficulties in reconciling—a labour cost theory of value and a labour commanded theory of value. The labour cost theory recognised that the value of a commodity was determined by the total amount of labour it took to produce that commodity—the embodied labour content. The labour commanded theory, on the other hand, stated that the value of a good was measured by the quantity of labour it was able to command or be exchanged for in the market place. Only under special circumstances would the embodied labour value equal the labour commanded value of a commodity, which presented particular theoretical problems for Smith. Nevertheless, Smith’s dilemma foreshadowed the split between the ‘production cost theories of value’ and the ‘subjective
preference theory of value’ which exist to this day in economics.

The labour theory of value, in its various guises, persisted in the classical period, with David Ricardo (1772–1823) in particular concentrating on removing the inconsistencies in the theory. Ricardo attempted to ‘prove’ the embodied labour theory of value, by demonstrating that the embodied labour content of a commodity provided an explanation of market prices. He was unsuccessful and settled for what Stigler (1965) coined a 93% labour theory of value, as embodied labour only explained 93% of the variation in market prices. Ricardo also recognised that the labour input theory of value would only hold if there was a constant capital:labour ratio in all sectors. If this assumption is dropped then the direct relationship between labour input and price cannot be guaranteed, or worse still there is an interdependence between income distribution and price. Ricardo recognised that the resolution to this problem was to find an invariable standard of value which is not affected by income distribution.

J.S. Mill (1806–1873) started the movement away from a strict embodied labour theory of value with his consideration of the idea of utility. Value in this context was seen to be determined by the ‘pleasure’ that can be derived from a product. Mill directly derived this idea of value equalling utility from Jeremy Bentham, the founding father of ‘utilitarianism’. This line of thinking was taken further by Menger (1840–1921), Walras (1834–1910) and Jevons (1835–1882) who all simultaneously derived the concept of diminishing marginal utility—the idea that most pleasure is gained from consuming the first unit of product and progressively less pleasure is gained from consuming each subsequent unit of product.

The scene was now set for the reconciliation of ‘utility’ ideas of value with ‘cost’ ideas of value, within one coherent theory of value. The so-called ‘neoclassical revolution’ was about to take place. Alfred Marshall (1842–1924) achieved this by constructing his now well-known ‘Marshallian Scissors’, consisting of a supply curve (marginal costs) and a demand curve (marginal utility). The equilibrium price was found to be the point where the two curves intersected (i.e. marginal utility = marginal cost). At this equilibrium price, the maximum net economic benefit is generated when it is achieved across all commodities in the economy. This became the standard theory of value (price) that has dominated neoclassical economics to this day. It has become the orthodox approach—virtually unchallenged and widely applied to a whole range of public policy issues, including ecological problems.1

1.3. Theories of value in ecological economics

An embodied energy theory of value has frequently been proposed as being suitable for ecological economics (Lavine, 1984; Judson, 1989; Hall et al., 1992). Costanza (1981) for example, argues for “...a cost of production theory with all costs carried back to the solar energy necessary to produce them”. Energy is seen to be the fundamental factor that ‘drives’ the production or activity of all economic and ecological systems. To support this type of thesis, Cleveland et al. (1984) produce empirical evidence to demonstrate how economic activity (GDP) is highly correlated with energy inputs. Odum (1983, 1996) takes this argument one step further by stressing the long-term importance of the maximum power principle. It is suggested, according to this principle, that eventually systems which transform energy at the most optimal rate will out-compete other systems and therefore survive. On this basis, Odum (1996) proposes an energy theory of value based on the idea of EMERGY—that is, the measure of the value of a commodity is the amount of energy required to produce it. This approach has recently been critically reviewed by Brown and Herendeen (1996).

This debate on the appropriateness of an energy theory of value came to a head with the

1 The only serious challenge to the neoclassical theory of value has been presented by Sraffa (1960) who built on Ricardo’s earlier work during the classical period. Sraffa’s (1960) determination of price does not depend on marginal analysis, rather it depends on solving a system of simultaneous equations in order to reveal the price of commodities. Sraffa’s (1960) theory is critically reviewed as a candidate theory of value for ecological economics in Section 3 of this paper.
Georgescu-Roegen (1979) suggestion that ‘matter matters too’. He rejected a strict energy theory of value, arguing that matter is also subject to the entropy law just as energy is and therefore energy should not assume supremacy in any physical view of value.

A central problem with operationalising any theory of value in ecological economics is the ‘mixed-units’ or ‘valuation’ problem. Whether you are dealing with solely energy flows or perhaps energy and mass flows, inputs and outputs need to be commensurated with each other. Analytical enquiries into this problem have led to a more precise theory of value for ecological economics, whereby ‘prices’ (value weights) can be determined for the biophysical inputs and outputs in the system of interest. Costanza and Hannon (1989) tackled the ‘mixed units problem’ by quantifying essentially mass and energy flows in the biosphere. By using matrix inversion techniques, they determined ‘prices’ (value weights) for various biosphere commodities. Patterson (1983) approached the ‘mixed units problem’ by analysing energy flows in economic systems. This enabled the determination of ‘prices’ (quality coefficients) for each energy input/output in the system, by solving a system of simultaneous linear equations. The Patterson (1983) theory is essentially one of a pure embodied energy theory of value, as unlike Costanza and Hannon (1989), there is no direct determination of price for mass inputs/outputs.

Contributory value is another concept which has been recognised in ecological economics. This idea has been promoted by Norton (1986) and Ulanowicz (1991), who attempt to quantify the indirect contributions made by different ecosystem components to the system’s final output. This again is a ‘cost of production’ approach which is similar to the energy theory of value approach and that of the classical economists. The contributory value idea is useful in that it makes ‘visible’ the contribution of ecosystem components to supporting economic and human activities. It cannot be considered a fully-fledged theory of value as it is unclear exactly how these contributory values can be quantified, or as Costanza (1991) puts it, these values have ‘remained empirically elusive’. The idea of contributory value may however be able to be integrated with the emerging theories of value in ecological economics such as those discussed in this paper.

2. Accounting basis for theories of value in ecological economics

Accounting frameworks are more than just a set of bookkeeping rules and conventions. They represent a particular conceptualisation or worldview of how the economy and ecological systems operate. If the accounting framework that is applied to an economic or ecological system is founded on questionable or inappropriate concepts, then it follows that the ‘prices’ or ‘values’ derived from such a framework are also of questionable validity. Attention, therefore, needs to clearly focus on what constitutes an appropriate accounting framework for ecological economics, before any set of prices (value weights) can be implied from the data.

Sahu and Nayak (1994) assert that most ecological economists view the economy as a biophysical system, operating just the same way as ecosystems do in terms of their adherence to thermodynamic and ecological principles. Within the context of the biophysical perspective of the economy, the logical accounting system is one based on energy and mass flows, rather than the measurement of exchange values as used in neoclassical economics. The theoretical rationale for mass/energy accounting of the economy rests on the work of Kneese et al. (1970), Georgescu-Roegen (1971) and Daly (1973) who all promulgated this biophysical view of the functioning of the economy in the early 1970’s. The basic conceptualisation is of an open economic system drawing on low entropy matter/energy, which is then dissipated as an output of high entropy matter/energy. Through the 1970’s and 1980’s, empirical work based on this theoretical perspective, concentrated in particular on tracing the flow of energy in economic systems through using input–output analytics (Bullard and Herendeen, 1975) as well as simulation modelling (Odum and Odum, 1976). Data difficulties have, however, meant that the characterisation of economic systems in terms of
mass flows has been mainly limited to individual economic processes and industries (Ayres, 1978). Though recently, some countries, as reported by Adriaanse et al. (1997), have quantified mass flows on an economy-wide basis and in addition, a full scale physical input–output matrix of mass flows in the German economy has been constructed (Stahmer et al., 1996).

Mass/energy accounting has found a more comfortable home in conventional ecology, where the focus is on ‘natural’ ecosystems operating outside the normal sphere of the market economy. Ecologists have quantified energy flows in a wide range of ecosystems: salt marshes (Asmus and McKellar, 1989); marine (Wulff et al., 1989); aquatic (Hannon, 1979) and soil (Richards, 1974). Mass accounting is usually limited to one tracer element such as carbon, phosphorus or nitrogen. According to Ulanowicz (1991) the use of two or more tracer elements is much less common and is restricted to studies such as that by Fasham (1985), who simultaneously traced carbon and nitrogen flows in the marine euphotic zone.

2.1. Energy and mass conservation

The first principle that must be adhered to, once mass/energy accounting is accepted as the correct theoretical basis, is that of energy conservation as prescribed by the First Law of Thermodynamics. The First Law of Thermodynamics states that energy can neither be created nor destroyed, just converted from one form to another. That is, the energy input into all economic and ecological processes must equal the energy output from those processes. Frequently, in neoclassical accounting systems, some inputs (e.g. solar energy) and some outputs (e.g. low temperature heat) are systematically ignored, in violation of the First Law of Thermodynamics. In an analogous fashion, mass inputs and outputs into ecological and economic processes must always be conserved, according to the Materials Balance Principle espoused by Kneese et al. (1970). Neoclassical models frequently ignore the Mass Balance Principle by not recognising that economic systems feed off flows of matter which are always finite and must always be deposited back into the environment or recycled. Indeed, very few economy–environment models or accounting frameworks explicitly take account of either the mass or energy balance principles, with Victor’s (1972) input–output modelling framework being one of the few exceptions.

2.2. Open systems

Ecological economic accounting frameworks can be applied to the biosphere, ecosystems, other ecological processes, the economy or economic sectors. From a mass/energy accounting standpoint, all of these systems must be open systems, in that they feed on inputs of energy and mass from the environment and reject wastes and other products to the environment. From this biophysical perspective, a closed system, where there are no external inputs/outputs of energy and mass, is a physical impossibility—such systems are perpetual motion machines as highlighted by Georgescu-Roegen (1971) among others. Costanza (1981) points out that the biosphere requires large inputs of solar energy to drive the circulation of matter via biogeochemical cycles, with negligible inputs of other forms of energy or mass. This solar energy is eventually degraded in accordance with the Second Law of Thermodynamics and is radiated as an output from the biosphere in the form of far-infrared energy. Ecosystems are similarly open systems with respect to energy flow, but more self-contained in terms of their matter flow via the functioning of biogeochemical cycles. Economic systems, both at the macro- and micro-level, are open systems drawing on natural resources from the environment and depositing pollutants and waste heat back into the environment. Superimposed on this pattern of mass/energy flows to and from the biophysical environment, economic systems are also open systems with respect to the flow of commodities as exports and imports from other economies.

2.3. Linear flow with feedbacks

As Daly (1985) argues, neoclassical models of the economy are based on the circular flow of
exchange value. Many economic textbooks unquestionably portray the circular flow model of the economy, with goods and services and factors of production flowing in a counter-current fashion to the flow of money. Heilbroner and Thirou (1981) assert that the ‘flow of output is circular, self renewing and self feeding’. From a biophysical perspective of mass and energy flows, this is a nonsense as it implies a closed system which is a thermodynamic impossibility.

Daly (1973, 1985) and Gilliland (1977) have shown that all economic systems are fundamentally linear and unidirectional in terms of their energy and matter flow patterns, starting with the depletion of low entropy natural resources from the environment and ending with the rejection of high entropy wastes back into the environment. Recycling can provide for some recirculation of matter with two important provisos. Firstly, as Georgescu-Roegen (1979) points out using iron data from Brown (1954), matter recycling can never be 100%. Secondly, all matter recycling necessarily requires extra inputs of energy—for instance, in transporting scrap iron to the smelter for melting down. Therefore, if there are technological, physical, economic and other constraints on energy supply, this will in turn impact on the feasibility of recycling matter.

It also needs to be recognised that energy, unlike matter, cannot be recycled once it has been dissipated from economic systems as low quality heat. Hence, energy flow will always be linear and unidirectional. Therefore it can be concluded that the flow of matter/energy through the economic system is one of linear throughput, with some minor feedbacks. As Daly (1985) aptly puts it, energy and matter flow is just like the ‘flow of a river which is ultimately linear though it may contain oxbows, whirlpools and eddies’.

### 2.4. Ecological flows should not be hidden

Many accounting models of economy–environment interactions typically treat ecosystem services and outputs as ‘free gifts’ that are magically donated by nature. These services and outputs just appear and according to the neoclassical doctrine can be ascribed value by employing non-market valuation techniques such as contingent valuation procedures. In this circumstance, human beings are asked via the WTP or WTA surveys to place a subjective value on the end-product of an ecological process. Costanza (1991) however, shows that this can lead to anomalies based on human beings having imperfect knowledge of ecological processes and functions. For example, he points out that human beings generally assign higher value to species of direct commercial value and/or species that are easy to empathise with such as mammals like dolphins, whales, giant pandas or gorillas. Invertebrates, on the other hand, because they are relatively ‘invisible’ and their role in the food chain is not well understood, tend to receive lower valuations in contingent valuation surveys.

Ulanowicz (1991) argues that the concept of ‘contributory value’ should be used, in order to overcome the type of problems encountered in neoclassical valuation techniques. Contributory value takes account of the direct and indirect mass and energy inputs required to produce an ecosystem product or service. Very few economy–environment accounting frameworks take explicit account of the biological and geological processes that provide natural resources to the economy, as well as absorb the emissions and pollutants rejected from the economy. One such example of an accounting framework is the Isard (1969) model, where ecological processes are quantified in Quadrant IV. Isard (1969) demonstrates the operationalisation of this model by quantifying in energy and mass terms the food chain required to produce cod.

### 2.5. Joint production, interdependencies and complex systems

Ecological and economic systems, when described in terms of mass and energy flows, are complex systems. Each process has multiple inputs and multiple outputs—joint products are the rule rather than the exception. The processes are all interdependent on each other, usually with a complex array of interlinked inputs and outputs. Many accounting frameworks used in neoclassical economics and ecology (particularly those based
on Leontief (1982) input–output analysis), oversimplify matters assuming that there is only one homogenous output per process (sector) and that the number of processes equals the number of commodities. Although these assumptions simplify the mathematical manipulation of data by allowing matrix algebra techniques to be used, they do not truly depict the functioning of complex ecological or economic systems. Often a linear functional form is also assumed, which may be appropriate for static models, but is not suitable for dynamic applications. Therefore, accounting frameworks need to allow for joint production, interdependencies, possible overdeterminancy (more processes than commodities) and non-linearities in dynamic applications, before any prices (values) can be imputed from the data.

3. Sraffa’s theory of value

Sraffa (1960) in his landmark monograph ‘production of commodities by means of commodities’ challenged the neoclassical theory of value (price) by producing an alternative model of price determination. Sraffa’s approach resurrected and then refined the Ricardian and classical ideas of value and distribution, by positioning them in an internally consistent mathematical framework. Judson (1989) argues that Sraffa’s theory of value is ‘very similar’ to the embodied energy theory of value and as such it could provide a suitable theory of value for ecological economics.\(^2\)

The Judson (1989) argument now needs to be critically re-examined particularly in terms of the ecological and thermodynamic principles outlined in the previous section of this paper.

3.1. Price determination in the Sraffa model

Sraffa (1960) essentially outlines two types of models of the economy—a ‘subsistence’ economy that produces no surplus and a ‘surplus’ economy. Both models involve the construction of a system of simultaneous linear equations that describe the exchange of commodities in the economy. These equations are solved to determine the relative prices (values) of each commodity in the economy.

To be specific, the Sraffa (1960) notional subsistence economy consisted of the following exchange transactions:

\[
\begin{align*}
240 \text{ qr wheat} + 12 \text{ tons iron} + 18 \text{ pigs} & \rightarrow 450 \text{ qr wheat} \\
90 \text{ qr wheat} + 6 \text{ tons iron} + 12 \text{ pigs} & \rightarrow 21 \text{ tons iron} \\
120 \text{ qr wheat} + 3 \text{ tons iron} + 30 \text{ pigs} & \rightarrow 60 \text{ pigs}
\end{align*}
\]

In this situation, there is no surplus generated by the economy, as there is no excess of wheat, iron or pigs.

These exchanges can be represented by a system of linear equations:

\[
\begin{align*}
240 \beta_1 + 12 \beta_2 + 18 \beta_3 &= 450 \beta_1 \\
90 \beta_1 + 6 \beta_2 + 12 \beta_3 &= 21 \beta_2 \\
120 \beta_1 + 3 \beta_2 + 30 \beta_3 &= 60 \beta_3
\end{align*}
\]

(1)

Any one of the commodities in the system can be used as the numeraire in order to solve these equations. If wheat is used as the numeraire (\(\beta_1 = 1\)), then the solution to these equations is:

\[
\begin{align*}
\beta_1 &= 1 \\
\beta_2 &= 10 \\
\beta_3 &= 5
\end{align*}
\]

These solution factors can be used to set the prices (values) of commodities relative to the numeraire. In this example, the ‘price’ of 1 ton of iron is 10 quarters of wheat and the ‘price’ of one pig is 5 quarters of wheat. It should be noted that irrespective of what commodity is used as the numeraire, the price relativities remain constant.

Sraffa (1960) then considered an economy that produces a ‘surplus’ and all of this surplus is distributed to profits (capitalists). Given this situation, his two commodity economy is:

\[
\begin{align*}
280 \text{ qr wheat} + 12 \text{ tons iron} & \rightarrow 575 \text{ qr wheat} \\
120 \text{ qr wheat} + 8 \text{ tons iron} & \rightarrow 20 \text{ tons iron}
\end{align*}
\]

\(^2\) England (1986) makes a similar proposition in suggesting that the Sraffian model, unlike the neoclassical model, can be readily reinterpreted from an ecological perspective.
In this notional example, the economy has a surplus of 175 quarters of wheat with no deficit or surplus of iron. If all of the surplus is allocated to profits the price of the commodities is determined by the following equations:

\[(280 \beta_1 + 12 \beta_2)(1 + r) = 575 \beta_1\]
\[(120 \beta_1 + 8 \beta_2)(1 + r) = 20 \beta_2\]  \hspace{1cm} (2)

where, \( r \) = profit rate expressed as a proportion.

The profit rate is calculated by determining the maximum or dominant eigenvalue, of the following matrix:

\[
\begin{bmatrix}
280 & 12 \\
575 & 20 \\
120 & 8 \\
575 & 20
\end{bmatrix}
\]  \hspace{1cm} (3)

The profit rate \((r)\) is related to the maximum eigenvalue \((\lambda_{\text{max}})\) by the following equation:

\[r = \left( \frac{1}{\lambda_{\text{max}}} \right) - 1\]  \hspace{1cm} (4)

Once the maximum eigenvalue is determined using convergence methods, as outlined by Ralston (1965), the equations can be solved to determine the prices using standard matrix algebra techniques such as matrix inversion. In this case:

\[\lambda_{\text{max}} = 0.80\]
\[\beta_1 = 1\]
\[\beta_2 = 15\]

This means there is a profit rate for each process (sector) of 25%, i.e. \(((1/0.80) - 1) \times 100\) and the ‘price’ of 1 ton of iron is 15 quarters of wheat.

There are two important assumptions in the Sraffa (1960) model where the surplus is entirely distributed to profits: (1) The profit rate is the same across all processes (sectors) in the economy. It is implicitly supposed that in the long-run profit ratios are equalised as an ‘equilibrium’ position is attained. (2) That the maximum eigenvalue is the appropriate eigenvalue. Other eigenvalues exist that will enable Eq. (3) to be solved, but they give rise to negative prices. These eigenvalues are therefore discarded because negative prices are considered to be ‘meaningless in economic terms’.

Next, Sraffa (1960) considers the situation where all of the ‘surplus’ is distributed to wages (labour) and each sector is assumed to have a zero rate of profit. This involves including the labour inputs into the model which had previously been assumed to be zero in the profit maximisation model:

\[
280 \text{ qr wheat} + 12 \text{ tons iron} + 100 \text{ person-hours} \rightarrow 575 \text{ qr wheat}
\]
\[
120 \text{ qr wheat} + 8 \text{ tons iron} + 50 \text{ person-hours} \rightarrow 20 \text{ tons iron}
\]

In algebraic notation, these exchanges can be represented by the following system of homogeneous linear equations, where outputs are treated as positive elements and inputs as negative elements:

\[
295 \beta_1 - 12 \beta_2 - 100 l = 0
\]
\[120 \beta_1 + 12 \beta_2 - 50 l = 0\]  \hspace{1cm} (5)

To determine the ‘price’ of the commodities and the wage rate (‘price’ of labour), a final equation needs to be inserted. This equation is the sum of the column elements in Eq. (5). Hence, we have:

\[
295 \beta_1 - 12 \beta_2 - 100 l = 0
\]
\[120 \beta_1 + 12 \beta_2 - 50 l = 0\]
\[175 \beta_1 + 0 \beta_2 - 150 l = 0\]  \hspace{1cm} (6)

Essentially this final equation allocates all of the surplus to labour and makes the system equations determined (i.e. df = 0). The solution of this system of simultaneous equations reveals the price of the commodities and labour:

\[\beta_1 = 1.00\]
\[\beta_2 = 14.86\]
\[l = 1.17\]

Sraffa (1960) recognises that these are two extreme situations of allocating all of the surplus either to profits or to wages and there are an infinite number of intermediate cases that can be represented on a wage-profit tradeoff curve.
3.2. Ecological critique of Sraffa’s theory of value

Both England (1986) and Judson (1989) suggest that Sraffa’s theory of value is more appropriate in ecological economics than is neoclassical price theory and valuation procedures. From a biophysical perspective, which is the basis of ecological economics, this suggestion is questionable on a number of grounds:

1. Sraffa is concerned with exchange values instead of biophysical flows. The Sraffa model does not map physical flows of energy and mass, although the inputs and outputs are measured in physical terms. The processes are clearly exchange processes. For example, take the first process in Sraffa (1960) subsistence model:

\[
240 \text{ qr wheat} + 12 \text{ tons iron} + 18 \text{ pigs} \rightarrow 450 \text{ qr wheat}
\]

In the market exchange process one might exchange 240 qr wheat plus 12 tonnes of iron plus 18 pigs for 450 qr wheat; but one would not contemplate physically converting these commodities to wheat. In biophysical terms, wheat production can be measured as coupled flows of mass and energy, such as those flows portrayed by the following equations:

\[
\begin{align*}
\text{CO}_2 + \text{Nutrients} + \text{Fertilisers} & \rightarrow \text{Wheat Biomass} + \text{Pollutants} + \text{O}_2 \\
\text{Solar Energy} + \text{Fossil Fuels} & \rightarrow \text{Calorific Value of Wheat} + \text{Low Grade Heat}
\end{align*}
\]

These equations are, of course, oversimplified and do not contain the required level of detail with respect to matters such as stoichiometric balance.

2. Sraffa’s model is a ‘subjective’ rather than ‘objective’ theory of value. It is often claimed that Sraffa’s model represents an ‘objective cost theory of production’, as opposed to the neoclassical ‘subjective preference theory of value’. In light of the fact that Sraffa’s model is based on exchanges of commodities, rather than a quantification of the inputs physically required to produce a commodity, this claim can be seriously questioned. Each equation in the Sraffa system represents what one party is willing to give up in order to receive some other commodity. At its very essence, this is a subjective process, whereby the consumer/producer weighs up his/her own preferences based on his/her perception, tastes and knowledge. It is not an objective measure of what is required to physically produce the commodity. The Sraffa model in this respect, is little different to the neoclassical theories of value and one must not be misled just because Sraffa measures process inputs/outputs in physical units.

3. Sraffa’s model doesn’t allow for conservation of mass and energy. The Sraffa model does not explicitly adhere to the principles of conservation of mass and energy, either at the individual process level or at the system level. At the process level, no mechanism is put into place to ensure that \(\text{mass}_{\text{in}} = \text{mass}_{\text{out}}\) or \(\text{energy}_{\text{in}} = \text{energy}_{\text{out}}\). It is therefore doubtful if any of Sraffa’s exchange processes will ever meet these requirements, especially as the ‘waste’ products are not taken into account. In fact, there is no logical reason why Sraffa (1960) exchanges, (which are represented by equations), should observe the mass and energy conservation principles. At the system level, the issue becomes more transparently problematic when one examines the net inputs and outputs into Sraffa’s economies. Take, for example, Sraffa’s surplus economy as previously discussed in this paper:

\[
\begin{array}{c@{}c@{}c@{}c}
\text{Wheat} & \text{Iron} \\
\text{Process 1} & 295 & -12 \\
\text{Process 2} & -120 & 12 \\
\text{Net input/output} & 175 & 0
\end{array}
\]

This implies that a surplus of 175 qr of wheat is being produced from nothing—mass creation which is clearly a physical impossibility. Wheat in biophysical terms is produced from mass inputs of nutrients and \(\text{CO}_2\) and energy inputs of solar energy and fossil fuels. If the Sraffa (1960) model observed the mass/energy conservation principles, these input requirements would be made explicit, as would the fact that all the outputs (e.g. pollutants as well as wheat biomass) also need to be taken into account. Once the mass/energy conservation principles are made an integral part of the analysis, the dependence of wheat production on possibly unsustainable inputs becomes explicit, as
does the fact that pollutants are also an inevitable consequence of wheat production which again has sustainability implications.

4. Sraffa's model is based on circular flow not linear throughput. Once you accept that mass/energy conservation principles should be applied to each process, then it logically follows that at the macroscopic level linear throughput with feedbacks is inevitable. The circular flow model, on the other hand, more naturally results from the exchange-value model that is put forward by Sraffa (1960).

The very essence of the Sraffa (1960) model is circular flow. Without some circularity in the irreducible matrix, the system of simultaneous linear equations is not determined (i.e. number of unknowns does not equal the number of equations). Circularity can be guaranteed in a number of different ways, but one certain guarantee is the insertion of labour into the schema. Labour is both an input into the production process as well as being a consumer of commodities.

From a biophysical perspective the circular flow models are a physical impossibility, operating for all intents and purposes as a ‘perpetual motion’ machine. The type of model presented in Eq. (2) (from Sraffa, 1960) is a circular flow model that produces a surplus from nowhere. A biophysical characterisation of the economy could not result in such a model being permissible. Even though some biophysical processes, such as biogeochemical cycles, appear to be circular, closer examination shows this is not strictly the case. Biogeochemical cycles do indeed involve the circulation of mass, but require inputs of energy to ‘drive’ this circulation which inevitably leads to outputs of low quality heat, which is not circulated. Therefore, the biogeochemical cycle is strictly one of linear throughput of energy with circulation of matter. When constructing the system of equations, one cannot simply decouple the linear flow of energy from the circulation of matter, just for the sake of analytical convenience. Both energy flow and matter circulation are inextricably intertwined with each other, whether it be a biogeochemical cycle or the recycling of matter in the market economy.

5. Sraffa's model assumes determinancy. The Sraffa (1960) model is always presented as though there are the same number of process in the economy as there are commodities. This enables the construction of a uniquely determined system of linear equations, which makes the determination of prices straightforward based on standard matrix algebra techniques. Rarely, if ever, are there equal processes and commodities in the economy, as has been pointed out by many others (Schefold, 1978; Howard and King, 1985). From a biophysical perspective, it is even more likely that a complex economic–ecological system will have more processes than commodities—overdeterminacy is probable. This problem cannot be ignored as it is in the Sraffa (1960) stylised presentation of the facts. Solution methods need to be devised that acknowledge overdeterminacy, rather than assuming it away by employing an a priori assumption, such as those that suppose that only the most efficient process for producing a particular output should be included in the model.

4. Patterson's embodied energy theory of value

Judson (1989) argued that the embodied energy theory of value is very similar to the Sraffian system of price determination and furthermore that both frameworks constitute a suitable theory of value for ecological economics. It was demonstrated in Section 3 that Sraffa’s theory of value is not suitable for ecological economics because it violates a number of biophysical principles. Nevertheless, if there is a direct energy analogue of the Sraffian theory of value, it has to be the Patterson’s (1983, 1993) QEM. The point has been recognised by others (Peet and Baines, 1986) but not recognised by Judson (1989).

4.1. Price determination in the Patterson model

The starting point in the Patterson (1983) QEM is to describe the energy flows in the reference system which can be any economic or ecological system. For example, an energy matrix like that described by Table 1 can be constructed. Each process involves the conversion of one direct en-
Table 1
Hypothetical national energy system

<table>
<thead>
<tr>
<th>Processes</th>
<th>Energy commodities (PJ)</th>
<th>Hydro</th>
<th>Wellstream gas</th>
<th>Crude oil</th>
<th>Delivered electricity</th>
<th>Oil products</th>
<th>Delivered gas</th>
<th>Heat</th>
<th>Transport</th>
<th>Light</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Hydro-electricity → delivered electricity</td>
<td>−14.50</td>
<td>13.50</td>
<td></td>
<td></td>
<td>−0.10</td>
<td>−0.20</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Delivered gas → delivered electricity</td>
<td>2.00</td>
<td>−6.00</td>
<td></td>
<td></td>
<td>−0.80</td>
<td>−0.02</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Oil products → delivered electricity</td>
<td>0.50</td>
<td>−2.00</td>
<td></td>
<td></td>
<td>−0.80</td>
<td>−0.01</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Wellstream gas → delivered gas</td>
<td>−16.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>14.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Crude oil → oil products</td>
<td>−125.00</td>
<td>100.00</td>
<td></td>
<td></td>
<td>−0.20</td>
<td>−0.04</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. Delivered electricity → heat</td>
<td>−6.00</td>
<td></td>
<td></td>
<td></td>
<td>6.00</td>
<td>−0.03</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7. Delivered gas → heat</td>
<td>−4.00</td>
<td></td>
<td></td>
<td></td>
<td>3.00</td>
<td>−0.10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8. Oil products → heat</td>
<td>−8.00</td>
<td></td>
<td></td>
<td></td>
<td>4.80</td>
<td>−0.04</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9. Delivered gas → transport</td>
<td>−4.00</td>
<td></td>
<td></td>
<td></td>
<td>−0.04</td>
<td>0.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10. Oil products → transport</td>
<td>−80.00</td>
<td></td>
<td></td>
<td></td>
<td>−0.04</td>
<td>0.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11. Delivered electricity → lighting</td>
<td>−10.00</td>
<td></td>
<td></td>
<td></td>
<td>−0.04</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Net input/output | −14.50 | −16.00 | −125.00 | 0.00 | 10.00 | 0.00 | 12.62 | 8.16 | 1.00 |

Direct energy inputs and outputs are indicated in normal type face. Indirect (embodied) energy inputs are indicated in italics.
ergy input into usually one direct output—e.g. electricity to heat. There are also indirect energy inputs required for most processes—e.g. indirect energy is required to physically produce an electric heater, such as the energy required to produce the steel componentry. In other words, there is embodied energy required to supply any given energy output.

The next step is to convert the reference system matrix, to a system of simultaneous linear equations:

\[(T - R)\beta + e = 0\]  \(\text{(7)}\)

where: \(T\), outputs matrix \([m \times n]\) representing direct energy outputs from 1…n for each process \(m\); \(R\), inputs matrix \([m \times n]\) representing direct and indirect energy inputs from 1…n, for each process \(m\); \(\beta\), solution vector \([n \times 1]\) representing the quality coefficients (‘prices’) for each energy type; \(e\), residuals vector \([m \times 1]\).

These equations can be solved by setting any one of the quality coefficients to unity. For example, using the Table 1 data, the following quality coefficients (‘prices’) are obtained by setting hydroelectricity to unity:

\[
\begin{align*}
    \beta_1 &= 1.00 \text{ (hydro-electricity) (by definition)} \\
    \beta_2 &= 0.40 \text{ (well stream gas)} \\
    \beta_3 &= 0.26 \text{ (crude oil)} \\
    \beta_4 &= 1.06 \text{ (delivered electricity)} \\
    \beta_5 &= 0.33 \text{ (oil products)} \\
    \beta_6 &= 0.46 \text{ (delivered gas)} \\
    \beta_7 &= 0.82 \text{ (heat)} \\
    \beta_8 &= 3.31 \text{ (transport)} \\
    \beta_9 &= 10.68 \text{ (lighting)}
\end{align*}
\]

Anyone of the specific coefficients in the reference system can be used as the quality equivalent unit numeraire. For a properly specified system of equations, it does not matter which energy type is used as the quality equivalent unit because the relativities between the quality coefficients remain constant.

4.2. Determination of process efficiencies and residuals

Besides the price vector \(\beta\), the other important information to be gained by solving the reference system equations is a measurement of the relative efficiency \(\Phi\) of each process in the system. The inclusion of the residual vector \(e\) in the equations is important as it acknowledges that all processes are not necessarily equally efficient and it furthermore enables the relative efficiency of each process to be calculated. The relative efficiency \(\Phi\) for each process can be calculated by dividing the process outputs (in quality equivalent units) by the process inputs (in quality equivalent units). For example, for the processes described in Table 1 the following relative efficiencies and residuals can be calculated:

<table>
<thead>
<tr>
<th>Process</th>
<th>Relative efficiencies</th>
<th>Residuals</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Hydroelectricity → delivered electricity</td>
<td>(\Phi_1 = 1.0000)</td>
<td>(e_1 = 0)</td>
</tr>
<tr>
<td>2 Delivered gas → delivered electricity</td>
<td>(\Phi_2 = 0.7544)</td>
<td>(e_2 = 0.6512)</td>
</tr>
<tr>
<td>3 Oil products → delivered electricity</td>
<td>(\Phi_3 = 0.3885)</td>
<td>(e_3 = 0.7869)</td>
</tr>
<tr>
<td>4 Wellstream gas → delivered gas</td>
<td>(\Phi_4 = 1.0000)</td>
<td>(e_4 = 0)</td>
</tr>
<tr>
<td>5 Crude oil → oil products</td>
<td>(\Phi_5 = 1.0000)</td>
<td>(e_5 = 0)</td>
</tr>
<tr>
<td>6 Delivered electricity → heat</td>
<td>(\Phi_6 = 0.7652)</td>
<td>(e_6 = 1.4381)</td>
</tr>
<tr>
<td>7 Delivered gas → heat</td>
<td>(\Phi_7 = 1.2879)</td>
<td>(e_7 = -0.5239)</td>
</tr>
<tr>
<td>8 Oil products → heat</td>
<td>(\Phi_8 = 1.3224)</td>
<td>(e_8 = -0.9142)</td>
</tr>
<tr>
<td>9 Delivered gas → transport</td>
<td>(\Phi_9 = 1.0725)</td>
<td>(e_9 = -0.1273)</td>
</tr>
<tr>
<td>10 Oil products → transport</td>
<td>(\Phi_{10} = 0.9950)</td>
<td>(e_{10} = 0.1273)</td>
</tr>
<tr>
<td>11 Delivered electricity → lighting</td>
<td>(\Phi_{11} = 1.0000)</td>
<td>(e_{11} = 0)</td>
</tr>
</tbody>
</table>

Processes that have a relative efficiency greater than 1 (\(\Phi > 1\)) are more efficient than the systems’
average, whereas processes that have a relative efficiency less than 1 ($\Phi < 1$) are less efficient than the system’s average. It can also be noted from this illustrative example that the relative efficiency of some processes equals 1 ($\Phi = 1$), which is the system’s average. This is, for the simple reason, that each of these processes are the only process either producing a direct output or using a direct input—they have no other competitive processes.

4.3. Contextualising Patterson’s QEM as a theory of value

The QEM was originally applied to commensurating only energy flows in complex economic systems. It does, however, have some striking similarities with Sraffa’s system, in the sense that energy value (price) is determined by solving a system of simultaneous linear equations. The most fundamental difference between the Patterson and Sraffa schema is that the former only measures energy flows and the latter measures exchange transactions in physical units.

From a biophysical perspective, the strength of the QEM is that it adheres to the accounting principles that are set out in earlier sections of this paper. It is a linear flow system with feedbacks (of indirect energy); it is an open system with net inputs and outputs crossing the system boundaries; and it is consistent with the energy conservation principle. The latter needs some explanation. If one compares the energy inputs and outputs of each of the processes in Table 1, one apparently does not get an equality which could imply a violation of the energy conservation law—the energy inputs are greater than the energy outputs. To reconcile this point, the reference system equations need to be written more explicitly:

$$(T - R)\beta + w + e = 0 \quad (8)$$

where: $w$, waste vector ($m \times 1$) of direct and indirect ‘waste’ heat of each process.

Now energy inputs ($\Delta H$) and energy outputs ($\Delta H$) of each process equal each other. Furthermore, if we solve the Eq. (8) we get a solution vector of $\beta = 1$ and $e = 0$, which tells us nothing more than that the First Law of Thermodynamics is being observed. The reference system equations are, however, solved by assuming that waste heat has no value and therefore $w = 0$. Hence, we have:

$$(T - R)\beta + e = 0 \quad (9)$$

The QEM model is also useful (in comparison to the Sraffa and the Costanza and Hannon models) in that it explicitly takes account of the fact that there can be more processes than commodities. This then leads to the insight that some processes are almost always more efficient than others as revealed by their relative efficiencies, rather than making the a priori assumption that they all have the same efficiency. This enables the analyst to pinpoint the most energy efficient processes, which can provide useful information to guide policy decisions and energy efficiency initiatives.

The limitation of the QEM as it has thus far been operationalised is that it only considers energy inputs and outputs. Mass flows are only important in the QEM insofar as they enable the indirect (embodied) energy inputs to be calculated. It could be considered in this context that the mass flows convey information about energy inputs into the system or process, but they are not measured directly nor are their relative prices (values) imputed in solving the equations. However, in principle, there is no reason why mass flows could not be included in the QEM:

$$(T + G - S - F)\beta + e = 0 \quad (10)$$

where: $T$, direct energy output matrix; $S$, direct energy inputs matrix; $G$, direct mass outputs matrix; and $F$, direct mass inputs matrix.

5. Costanza and Hannon’s theory of value

Costanza and Hannon (1989) describe an analytical framework for determining ‘prices’ for ecological commodities. The focus of their paper is similar to that of Patterson (1983), in that Costanza and Hannon (1989) structure the problem in terms of

---

3 Exactly the same approach of assuming the waste vector to be zero is used in the Costanza and Hannon (1989) model of price determination. Interestingly, they made the same point concerning the necessity for $\beta = 1$ (i.e. all prices equal unity) when the waste vector is included in their model because of the mass/energy conservation principles.
trying to solve the ‘mixed units’ or ‘apples and oranges’ problem. That is, the problem of commensurating essentially ‘dissimilar components’.

5.1. Price determination in the Costanza and Hannon model

Costanza and Hannon (1989) demonstrate how prices can be determined in ecological systems by applying their methodology to the biosphere, although its application to other ecological systems is also discussed. Their aggregated model of the biosphere involves nine commodities (manufactured goods, agricultural products, natural products, nitrogen, carbon, phosphorus, water vapour, fresh water and fossil fuels) and nine processes (urban economy, agriculture, natural plants, animals, soil, deep ocean, surface ocean, atmosphere and deep geology). The model is formulated in terms of a use matrix \( U \) which describes the inputs into each process in the biosphere, as well as a make matrix \( V \) which describes the outputs from each process in the biosphere (refer to Table 2). A vector \( z \) of net inputs is also used in the model. Essentially \( z \) is the input of solar energy into the biosphere, as Costanza and Hannon (1989) consider solar energy to be, for all intents and purposes, the only net input into that system.

The \( U \) and \( V \) matrices and the \( z \) vector form the basis of the Costanza and Hannon (1989) determination of ecological prices or weighting factors. The price determination is accordingly based on solving the following system of equations:

\[
\boldsymbol{\beta}^t U + z = \boldsymbol{\beta}^t V
\]

where: \( V \), make matrix \([n \times m]\) representing the outputs in the ecological system for each process; \( U \), use matrix \([n \times m]\) representing the inputs in the ecological system for each process; \( z \), vector \([1 \times m]\) representing the input of solar energy into each process; \( \boldsymbol{\beta}^t \), solution vector \([1 \times n]\) representing the ‘price’ of each commodity, enumerated in terms of \( \Delta H \) solar/unit of commodity.\(^4\)

In solving these simultaneous equations it is automatically assumed that solar energy is the numéraire in the system. Hence, the equations are rearranged so that \( z \) is the dependent vector:

\[
z = \boldsymbol{\beta}^t (V - U)
\]

The ecological prices \( \boldsymbol{\beta}^t \) can then be determined by matrix inversion:

\[
\boldsymbol{\beta}^t = z(V - U)^{-1}
\]

In this way Costanza and Hannon (1989) determine ‘prices’ of each of the commodities in the biosphere system expressed in terms of units of solar energy.

5.2. A more generalised formulation of the Costanza and Hannon model

The Costanza and Hannon (1989) model seems to be based on the idea that solar energy input is the only physical external input into the biosphere. Certainly this is the argument put forward in the predecessor model promulgated by Costanza and Neill (1981). This assumption that solar energy is the only external input is questionable, with respect to the biosphere and furthermore it can be demonstrated to be unnecessary in the determination of ecological prices.

The assumption that solar energy is the only external input in the biosphere is demonstrably incorrect on a strict accounting basis, even though it can be viewed as a tenable approximating assumption because of the enormity of solar inputs in comparison to other physical inputs. There are other external inputs of energy into the biosphere, including the gravitational pull of the moon that drives tidal systems and geothermal energy derived from the earth’s crust. Mass inputs (e.g. meteorites) and outputs (e.g. satellites and spacecraft) also exist although admittedly negligible. There are also issues of spatial and time scales, with respect to the claim that solar energy is the only external solar input into the biosphere. Firstly, there is a time scale issue with respect to fossil fuels. If a time scale of a calendar year is used, then fossil fuels are an external input into the biosphere—this is explicitly revealed by the Costanza and Hannon (1989) \( V-U \) matrix. That

\(^4\) It is assumed in Costanza and Hannon’s (1989) model that the number of processes \( m \) is the same as the number of commodities \( n \), hence \( m = n \).
### Table 2
Costanza and Hannon’s biosphere model

<table>
<thead>
<tr>
<th>Commodity and Process</th>
<th>Urban economy</th>
<th>Agriculture</th>
<th>Natural plants</th>
<th>Animals</th>
<th>Soils</th>
<th>Deep ocean</th>
<th>Surface ocean</th>
<th>Atmosphere</th>
<th>Deep geology</th>
<th>Net input/output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufactured goods (US, 1972 $10^{12}$)</td>
<td>1.27</td>
<td>-0.08</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.19</td>
</tr>
<tr>
<td>Agricultural products ($10^{15}$ g dry wt)</td>
<td>-1.28</td>
<td>4.55</td>
<td>-3.27</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Natural products ($10^{15}$ g dry wt)</td>
<td>-1.18</td>
<td>163.4</td>
<td>-24</td>
<td>-103.4</td>
<td>-34.6</td>
<td></td>
<td></td>
<td>-0.16</td>
<td>0.06</td>
<td></td>
</tr>
<tr>
<td>Nitrogen ($10^{12}$ g N)</td>
<td>25</td>
<td>-31.4</td>
<td>-208</td>
<td>295</td>
<td>-153.1</td>
<td>14</td>
<td>58.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Carbon ($10^{15}$ g C)</td>
<td>5</td>
<td>-2.1</td>
<td>-73.4</td>
<td>14</td>
<td>46.5</td>
<td>12.3</td>
<td>-2.3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Phosphorus ($10^{15}$ g P)</td>
<td>1.6</td>
<td>-28.5</td>
<td>-1345.7</td>
<td>232.9</td>
<td>1161.1</td>
<td>-21</td>
<td></td>
<td>-0.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Water vapour (km$^3$)</td>
<td>79</td>
<td>5931</td>
<td>50741</td>
<td>14650</td>
<td>424700</td>
<td>-496100</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fresh water (km$^3$)</td>
<td>-79</td>
<td>-5661</td>
<td>-51226</td>
<td>-12434</td>
<td>-424700</td>
<td>496100</td>
<td>2000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fossil fuels ($10^{12}$ g C)</td>
<td>-5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.07</td>
</tr>
<tr>
<td>Sunlight* ($10^{18}$ kcal)</td>
<td></td>
<td></td>
<td>-23</td>
<td>-227</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-856</td>
</tr>
</tbody>
</table>

Derived from Costanza and Hannon (1989). Inputs are negative entries. Outputs are positive entries.

* This row equals the vector $z$. 
is, if the net input/output column is calculated for their model, then there is a resultant net input of $4.93 \times 10^{15}$ g (carbon) of fossil fuels and $856 \times 10^{18}$ kcal of solar energy. Only over long time horizons (millions of years) can the fossil fuel be converted back to solar energy. Secondly, there is a spatial scale issue. The Costanza and Hannon (1989) model can only really be applied to an ecological system which encompasses the entire biosphere because it can be reasonably assumed that there is only one significant external input in the form of solar energy. However, as Costanza and Hannon (1989) themselves point out, at the ecosystem level there will be more than one net input, which leads to problems in terms of the current way they formulate their model.

Fortunately however, it can be demonstrated that the Costanza and Hannon (1989) assumption that there is only one external input of solar energy is not necessary for the determination of ecological prices. The model can be formulated in a more generalised way to side-step the necessity for this assumption:

$$b_t(V - U) - \beta_{10}z = 0$$  \hspace{1cm} (14)

where: $b_t$, solution coefficients representing the price of each from $b_1$ to $b_9$; $b_{10}$ = solution coefficient representing the price of solar energy.

In the original Costanza and Hannon (1989) model it is an a priori assumption that $b_{10} = 1$ (i.e. solar energy is the numeraire). Hence we have:

$$b_t(V - U) - z = 0$$  \hspace{1cm} (15)

By rearrangement and matrix inversion the ecological prices can be determined by:

$$\beta' = z(V - U)^{-1}$$  \hspace{1cm} (16)

We therefore solve Eq. (16) by assuming that the solar energy coefficient equals unity ($b_{10} = 1$) and we get the following solution coefficients:

$$\beta_1 = 190.365 \times 10^{18} \text{ kcal solar energy/}10^{12} \text{ g of nitrogen}$$
$$\beta_2 = 0.630 \times 10^{18} \text{ kcal solar energy/}10^{12} \text{ g of nitrogen}$$
$$\beta_3 = 57.123 \times 10^{18} \text{ kcal solar energy/}10^{15} \text{ g of carbon}$$
$$\beta_4 = 5.167 \times 10^{18} \text{ kcal solar energy/}10^{12} \text{ g of phosphorus}$$
$$\beta_5 = 0.550 \times 10^{18} \text{ kcal solar energy/}10^{15} \text{ g (carbon) in fossil fuels}$$
$$\beta_6 = 1.0000 \times 10^{18} \text{ kcal solar energy}$$

We can also solve Eq. (14) by arbitrarily setting any of the other coefficients to unity in order to avoid the trivial solution of $\beta = 0$. For example, we can set $\beta_5 = 1$ and we generate the following coefficients:

$$\beta_1 = 3.3318 \times 10^{15} \text{ g carbon/}10^{12} \text{ manufactured goods}$$
$$\beta_2 = 0.2453 \times 10^{15} \text{ g carbon/}10^{15} \text{ g (dry weight) of agricultural products}$$
$$\beta_3 = 0.6854 \times 10^{15} \text{ g carbon/}10^{15} \text{ g (dry weight) of natural products}$$
$$\beta_4 = 0.0110 \times 10^{15} \text{ g carbon/}10^{12} \text{ g of nitrogen}$$
$$\beta_5 = 1.0000 \times 10^{15} \text{ g carbon/}10^{15} \text{ g carbon (by definition)}$$
$$\beta_6 = 0.0204 \times 10^{15} \text{ g carbon/}10^{12} \text{ g phosphorus}$$
$$\beta_7 = 0.0096 \times 10^{15} \text{ g carbon/km}^3 \text{ water vapour}$$
$$\beta_8 = 0.0096 \times 10^{15} \text{ g carbon/km}^3 \text{ fresh water}$$
$$\beta_9 = 1.6833 \times 10^{15} \text{ g carbon/}10^{15} \text{ g (carbon) fossil fuels}$$
$$\beta_{10} = 0.0175 \times 10^{15} \text{ g carbon/}10^{18} \text{ kcal solar energy}$$

The price relativities between the first set of solution coefficients (when $\beta_{10} = 1$) and the second set of solution coefficients (when $\beta_5 = 1$), are exactly the same. For example, if you take the ratio between $\beta_1$ and $\beta_2$ in both sets of solutions, it is exactly the same. In other words, it is irrelevant what particular commodity is used as the numeraire because the price relativities remain constant.
Hence, a generalised way of expressing the solution to Eq. (14) in terms of the biosphere model data is:

\[
\begin{align*}
\beta_1 &= 3.3318x \\
\beta_2 &= 0.2453x \\
\beta_3 &= 0.6854x \\
\beta_4 &= 0.0110x \\
\beta_5 &= 1.0000x \\
\beta_6 &= 0.0204x \\
\beta_7 &= 0.0096x \\
\beta_8 &= 0.0096x \\
\beta_9 &= 1.6833x \\
\beta_{10} &= 0.0175x
\end{align*}
\]

The trivial solution to the system of simultaneous linear equations is \( x = 0 \). Non-trivial solutions are obtained when \( x \) is a positive number. In matrix notation, the solution vector for the Costanza and Hannon (1989) model is therefore:

\[
\beta = E x
\]

where: \( E \), vector \((n \times 1)\) of quality equivalent units; \( x \), scalar \((1 \times 1)\) representing any positive number.

Some reconceptualisation of the Costanza and Hannon (1989) model is therefore required, to reconcile the model with the fact that price is defined by the one dimensional solution space \( \beta = E x \). Within this solution space it is conventional to set one of the \( \beta \) coefficients to unity and then to express all of the other coefficients as multiples of that coefficient. This is akin to the idea of ‘relative prices’ in neoclassical economics. It is the relativity between the prices of different commodities that is critical, not the ‘actual prices’.

5.3. Convergence of Patterson’s approach with Costanza and Hannon’s approach

The foregoing analysis of both the Patterson (1983) and the Costanza and Hannon (1989) approaches reveals some close similarities and points where the two approaches can converge if one accepts the fundamental energy and mass flow schema. The Patterson (1983) approach, as previously demonstrated, can be extended to encompass mass as well as energy flows:

\[
(T + G - S - F)\beta + w + e = 0
\]

This system of simultaneous linear equations represents a biophysical system of mass and energy flows. This system should meet a number of conditions:

1. conform to the mass and energy conservation principles, both at the process and system levels;
2. only actual physical flows of mass and energy must be represented in the system. None of the constituent processes should be formulated in terms of exchange transactions which almost inevitably violate the mass/energy conservation principles;
3. the mass and energy flows must follow a pattern of linear throughput with feedbacks, not a model of circular flow as in neoclassical economics and Neo-Ricardian economics;
4. as a corollary to (3), the system must be open with any number of net inputs and net outputs. There is no need to assume just one net input (solar energy) which was previously assumed by Costanza and Hannon (1989);
5. the flow of mass and energy must encompass both the market economy and the natural environment, recognising that flows of mass/energy in the economy must be sustained by natural flows;
6. more processes than commodities \((m > n)\) are permissible and to be expected in most complex economic and ecological systems. This condition of \( m > n \), leads to the recognition that some processes are likely to be more efficient than others and hence the existence of the \( e \) vector in Eq. (18);
7. joint production (multiple products) at the process level in economic and ecological systems is possible and to be expected.

If Eq. (18) is solved, a solution vector of \( \beta = 1 \) will result. That is, all of the commodities will have a price of unity. This is a confirmation of the energy and mass conservation principles, a point recognised by this author and Costanza and Hannon (1989). Once the waste vector is excluded from the Eq. (18), as it is deemed to have zero value, a solution is obtained where all the commodities have different prices. Hence, the follow-
The system of equations needs to be solved to determine the ecological prices:

\[(T + G - S - F)\beta + e = 0\]  (19)

The approach outlined above is certainly similar to the Sraffa (1960) approach, as is indicated by Judson’s (1989) arguments. The broad similarity between the Sraffa (1960), Patterson (1983) and Costanza and Hannon (1989) approaches is that price is determined by solving a system of simultaneous linear equations. These equations characterise the inputs and outputs of processes in the system of interest (economic or energy or ecological system). The reliance on solving simultaneous equations sets these approaches apart from the standard neoclassical theory of price determination which is based on supply and demand curves. Nevertheless, the Sraffa (1960) model falls well short of an appropriate theory of value (price) for ecological economics, as it is not based on a biophysical view of the economy. As such Sraffa’s (1960) model generally does not conform to the first five conditions specified above.

On the other hand, the Costanza and Hannon (1989) approach can be readily modified to conform to principles (1)–(7). The major adjustment firstly required is the recognition that any commodity can successfully be used as the numeraire and there is nothing special about solar energy. Secondly, the Costanza and Hannon (1989) approach can be extended to allow for the possibility of more than one net input to the system and the possibility of more processes than commodities (which allows for different process efficiencies).

6. Outstanding issues in formulating a theory of value for ecological economics

6.1. Matrix dimensions and solution methods

It cannot be realistically assumed, as it was in Sraffa’s (1960) original subsistence model, that the number of processes will fortuitously equal the number of commodities. In this situation, the system of equations is determined \((df = 0)\) and hence there is one unique set of prices. Nor can it be assumed that the system of equations is underdetermined by \(1\) \(df\) as it is in the Costanza and Hannon’s (1989) model and in Sraffa’s (1960) surplus economy model. By setting one of the extra commodities to unity in these systems, the system of equations becomes determined and again we have a unique set of prices.

Instead, in most complex economic and ecological systems, there is a likelihood that there will be more processes than commodities (i.e. the system of equations will be overdetermined). A number of solution methods have been developed by Patterson (1983, 1993) for solving overdetermined systems of equations. The methods so far developed have their limitations, particularly with respect to assumptions made about joint production and there is clearly a need for further research in this area.

6.2. An embodied energy theory of value?

A model of price determination, based on mass and energy flows, was proposed in this paper. The question that now arises is, does this model \((T + G - S - F)\beta + e = 0\) really constitute an embodied energy theory of value. The answer to this question fundamentally depends on firstly how you define an energy theory of value and to a lesser extent on the time scales and spatial scales being used.

An energy theory of value can first of all be defined on the basis that energy is the only source of value in the system (Type I definition). That is, energy is the only primary input into the system. On this basis, it is reasonable to conclude (ignoring negligible mass inputs) that the Costanza and Hannon (1989) biosphere model of price determination does constitute an energy theory of value. If however, you were applying the model to an ecosystem or a national economy where there are other external inputs into the system apart from energy, you could not conclude that there is an energy theory of value, according to the Type I definition. This is because, from a biophysical viewpoint, these systems have significant inputs of mass (minerals, water, atmospheric chemicals etc.), that cannot be ignored.
An energy theory of value can also be defined in terms of how well embodied energy content explains market values (prices) (Type II definition). Cleveland et al. (1984) used this type of approach, for example, in an attempt to prove that embodied energy inputs into the US economy and sectors determine the value of output. There is however, no logical reason why the model \((T + G - S - F)\beta + e = 0\) proposed in this paper, should conform to this Type II definition of an energy theory of value. Prices in this model are solely determined by the interdependencies between commodities as mapped by mass/energy flows and there is no compelling reason why these prices should directly correspond to market prices that are based on subjective preference. Such attempts to establish empirical evidence for an embodied energy theory of value are forlorn and reminiscent of Ricardo’s unsuccessful attempts to establish an embodied labour theory of value.

It is also important to note that the choice of numeraire is not related to your choice of theory of value. The two issues should not be confused as they often are. Even if you structure the model in terms of an energy theory of value, according to the Type I definition, this paper has demonstrated that any of the non-energy commodities can be satisfactorily used as the numeraire. That is, the numeraire is only there as a unit of account to measure the price relativities between commodities and any one of the commodities will establish exactly the same price relativities. The numeraire-commodity is merely an anchor point or reference point for the solution space \(\beta = Ex\).

In summary, perhaps we should label the \((T + G - S - F)\beta + e = 0\) model of price determination which has been established in this paper as a biophysical theory of value rather than an energy theory of value. Once you acknowledge the existence of mass flows, it is inevitable that a strict energy theory of value will be hard to establish, other than applying the Type I definition to the biosphere level. For all other systems of interest to ecological economics, it is impossible to establish either Type I or II energy theory of value definitions, other than axiomatically excluding all mass flows from the analytical boundary.

6.3. Need for empirical applications

Sraffa (1960) demonstrated the use of his theory of value and distribution by way of purely hypothetical systems of algebraic equations. The literature which has exploded since the publication of Sraffa’s (1960) ‘production of commodities by means of commodities’ has been purely theoretical and to this author’s knowledge there has been no empirical applications of the Sraffian model. This in part has led to practical issues, such as the overdeterminacy of equations (more processes than commodities) being largely ignored for convenience sake. As a result the Sraffian theory of value has perhaps been all the weaker than if the analysts had confronted this theoretical development using real data.

Thus far, Costanza and Hannon (1989) have applied their price determination model to aggregate processes for the biosphere mainly using mass and energy flows. Patterson (1983) applied the QEM price determination model to energy flows in the New Zealand economy and a hypothetical energy efficient economy. Fruitful theoretical conclusions have arisen from these empirical applications, which would probably have not emerged from a purely theoretical approach. Indeed, previously England (1986), Schefold (1989) and Judson (1989) have considered the possibility of using Sraffa-type models to determine ecological prices, using only theoretical discourse and perhaps it is by no coincidence they have not pinpointed some of the problems in applying such models to ecological systems.

It is therefore important that any future developments of a theory of value (price) based on mass/energy accounting, have a strong empirical focus directed towards analysing policy options. Without this focus, such a theory of value is bound to be drawn down a theoretical cul-de-sac in much the same way as Sraffa’s theory of value—in danger of producing hypothetical answers to hypothetical questions. The initial ap-
plication of the QEM to the New Zealand energy system has provided some very useful policy insights into the efficiency of energy conversion processes in the New Zealand economy.

6.4. Need for methodological pluralism

One of the emergent methodological themes in the ecological economics literature is the need for methodological pluralism and openness, spearheaded by the writings of Norgaard (1985). There is a danger that the proposed biophysical theory of value (price) can be seen as reductionistic and foreclosing other methodological options. One only has to look at neoclassical economics since the marginal revolution to see how this can happen. The intention, however, of this proposed theory of value (price) is only to devise an empirical mechanism for resolving the ‘mixed units’ problem, which is encountered when evaluating energy/mass flows in economic–ecological systems. Without such a mechanism it is impossible to compare the efficiency of processes and systems, as no numeraire exists. Evaluation is therefore impossible, which hinders policy advice and analysis of the policy options. In stating this, it is recognised that economic and environmental policy advice can be derived from using a whole range of techniques, methods and approaches. A monolithic approach based on a single theory of value is not being proposed. Neoclassical economics has fallen into this trap and ecological economics should avoid it.

References


